

(3 hours)

Total marks:80

N.B.: (1) Question No. 1 is compulsory.

(2) Attempt any Three from remaining.

- Q1 a) If $\tanh x = 1/2$ then find value of x and $\sinh 2x$ [3]
 b) If $u = \log(x^2 + y^2)$ Find $\frac{\partial^2 u}{\partial x \partial y}$ [3]
 c) If $x = u - uv$, $y = uv - uvw$, $z = uvw$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ [3]
 d) Using Maclaurin's series, Prove $\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} + \dots$ [3]
 e) Check if the matrix $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$ is unitary [4]
 f) Find n^{th} derivative of $\frac{2}{(x-1)(x-2)(x-3)}$ [4]
- Q2. a) Solve $x^5 = 1 + i$ and find the continued product of the roots. [6]
 b) Reduce the matrix $A = \begin{bmatrix} 3 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ to the normal form [6]
 and find its Rank
 c) State and Prove Euler's theorem for two variables hence [8]
 find value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ where $u = \frac{xy}{x+y}$
- Q3 a) Investigate for what values of λ and μ the equations [6]
 $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have
 i) No solution ii) Unique solution iii) Infinite solutions
 b) Examine the function for its extreme values [6]
 $f(x, y) = y^2 + 4xy + 3x^2 + x^3$
 c) If $\tan(\alpha + i\beta) = \sin(x + iy)$ then Prove $\frac{\tan x}{\tanh y} = \frac{\sin 2\alpha}{\sinh 2\beta}$ [8]
- Q4 a) If $x = u \cos v$, $y = u \sin v$ then [6]
 Prove $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)} = 1$
 b) Prove that $\log \left(\frac{\sin(x+iy)}{\sin(x-iy)} \right) = 2i \tan^{-1}(\cot x \tanh y)$ [6]
 c) Solve by Gauss Jordan method [8]
 $2x + 3y + 4z = 1$, $x + 5y + z = 1$, $x + y + 6z = 5$
- Q5. a) Prove $\cos^6 \theta - \sin^6 \theta = \frac{1}{32} [\cos 6\theta + 15 \cos 2\theta]$ [6]
 b) Evaluate $\lim_{x \rightarrow 0} \left[\frac{x - \sin x}{x^3} \right]$ [6]
 c) If $y = \cos(m \sin^{-1} x)$ then [8]
 prove that $(1 - x^2)y_{n+2} - (2n + 1)x y_{n+1} + (m^2 - n^2)y_n = 0$

Q6 a) Check if the following vectors

$$X_1 = [3, 1, 2, 1], X_2 = [4, 6, 2, -4], X_3 = [-6, 0, -3, -4]$$

$X_4 = [1, 0, 2, 1]$ are linear dependent hence find the relation between them if any.

[6]

b) If $f\left(\frac{z}{x^3}, \frac{y}{x}\right) = 0$ then

prove that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 3z$

[6]

c) Fit a second degree parabola $y = ax^2 + bx + c$ to the following data

[8]

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

(3 Hours)

Total Marks : 80

N.B.1. Question No. 1 is Compulsory.

2. Answer any Three more questions out of the remaining Five questions.

3. Assume any suitable data wherever required but justify the same.

4. Figures to the right indicate full mark

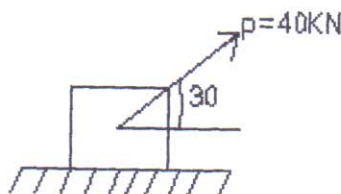
5. Take $g = 9.81 \text{ m/s}^2$

Q1. Attempt any four.

a) Replace four like parallel forces A,B,C,D of magnitudes 5N,10N,15N,20N respectively with 2m distance between each other by i) a force –couple system at the point of application of first force A. ii) a single resultant with respect to the first force A. [5]

b) State and prove Lami's Theorem. [5]

c) A block weighing 200KN is placed on rough horizontal plane. A pull force of 40KN is applied at an angle 30° on it. If the block is just on the point of moving, find Coefficient of friction and Angle of friction. [5]



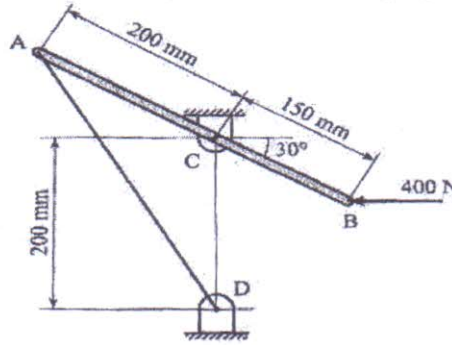
d) A car moves in a circular path of 2m radius. At an instant its speed is increasing at the rate of 6 m/s^2 and its total acceleration is 10 m/s^2 . Determine the speed of car at this instant. [5]

e) A car travelling at a speed of 60kmph is braked and comes to rest in 6 seconds after the brakes are applied. Find the minimum coefficient of friction between the wheels and the road. [5]

Q2a). A lever AB is hinged at C and attached to a cable at A. If the lever is subjected at B to a 400N horizontal force, determine

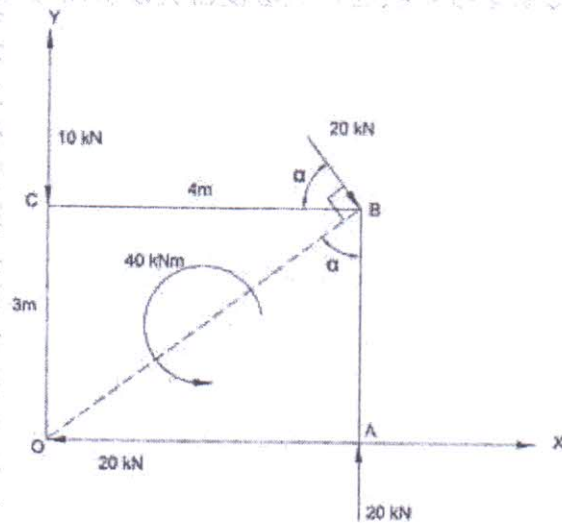
[8]

- (i) Tension in the cable AD (ii) The reaction at C.



b) Find the resultant of the force system acting on a body OABC, shown in figure. Also find the points where the resultant will cut the x and y axes. What is the distance of resultant from point 'O'?

[6]

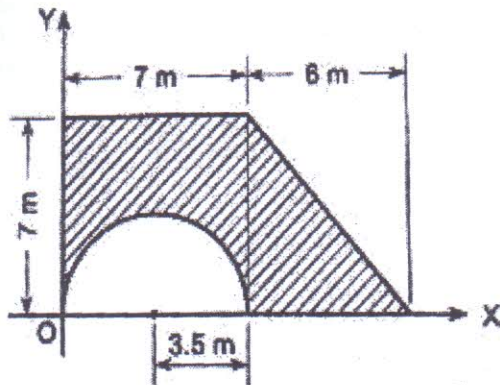


c) A boy throws a ball vertically downwards from a height of 1.5m. He wants to rebound it from floor and just touch the ceiling of room which is at a height of 4m from ground. If coefficient of restitution is 0.8. find the initial velocity with which the ball should be thrown.

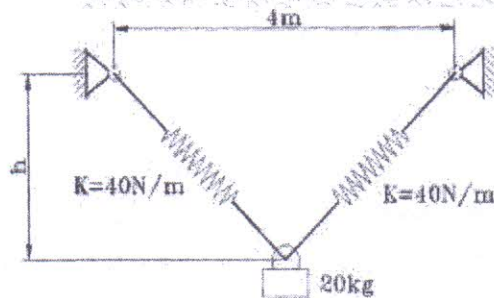
[6]

Q3.a) Determine the Centroid of the shaded area. Refer Fig(a)

[8]



Fig(a)



Fig(b)

b) A cylinder has mass 20 kg and is released from rest, when $h=0$. Determine the speed when $h=4$ m. The spring has an un-stretched length of 2 m. Refer Fig(b).

[6]

c) Replace the given force system by a force couple system at point P (0,2,1).

$F_1 = (12i + 16j + 3k)$ N acts at point A (0,0,3) m.

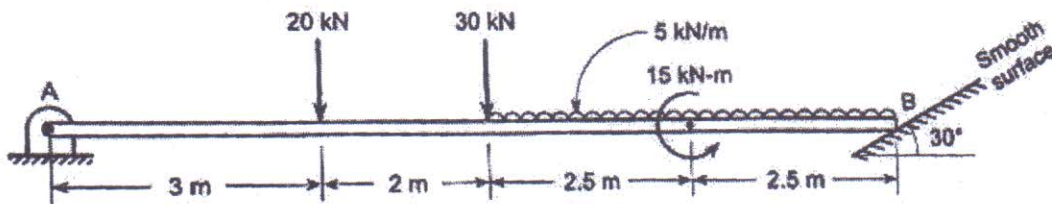
$F_2 = (7i + 6j)$ N acts at point B(3,-2,0) and a couple with moment

$M = (20k)$ N m.

[6]

Q4.a) Find support reactions at A and B for the beam loaded as shown in Fig(7).

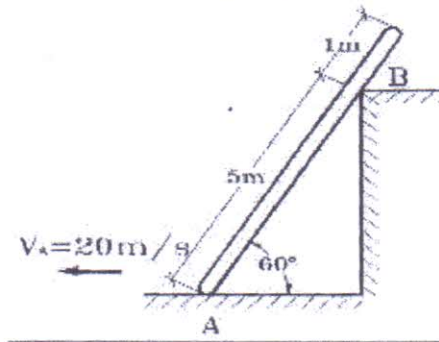
[8]



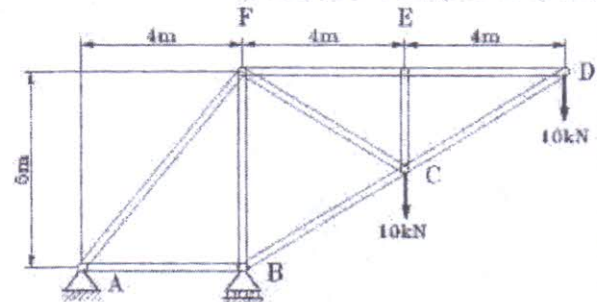
b) An aeroplane is flying in a horizontal direction with a velocity of 540 kmph at a height of 3000 m. When it is vertically above the point A on the ground, a box is dropped from it. The box strikes the ground at a point B. Calculate the distance AB on the ground. Neglect air resistance.

[6]

- c) Velocity of point A on rod is 20 m/s at the instant shown in Fig(c). Locate ICR for the rod and determine velocity of point B on the rod. [6]



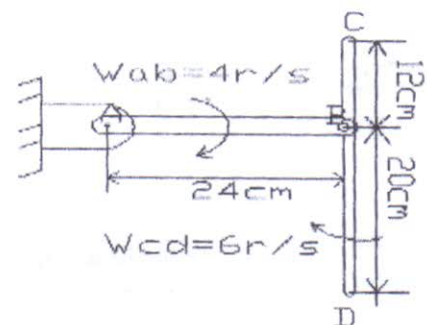
Fig(c)



Fig(d)

- Q5a) For the truss shown in Fig(d), determine

- (i) Forces in members EF, CF and BC by method of sections only. [3]
 - (ii) Forces in all other members by method of joints [5]
- b) A motorcycle starts from rest and accelerates at 2m/s^2 till velocity reaches 10m/s . Then it accelerates at 1m/s^2 till velocity reaches 15m/s and continues at uniform velocity of 15m/s till it covers a total distance of 300m . Find the total time taken to cover this distance. Draw the v-t and x-t graph for this motion. [6]
- c) A bar AB is hinged to wall at A. Another bar CD is connected to it by a pin at B such that $CB=12\text{cm}$ and $BD=20\text{cm}$. At the instant shown in Fig, AB is perpendicular to CD and the angular velocities of bar AB is 4rad/sec and that of CD is 6rad/sec both clockwise. Determine the linear velocities of points C and D. [Note:- bar CD is in plane motion] . [6]



- Q6 a) A uniform ladder of length 4m rests against a rough vertical wall with its lower end on a rough horizontal floor, the ladder being inclined at 50° to the horizontal. The coefficient of friction between the ladder and the wall is 0.3 and that between the ladder and the floor is 0.5 . A man of weight 500N ascends up the ladder. What is the maximum length up along the ladder, the man will

be able to ascend before the ladder commences to slip. The weight of the ladder is 1000N. [8]

b) Write down the conditions of equilibrium for Forces in Space for

(i) con-current system (ii) parallel system and (iii) general system. [6]

c) The velocity of a particle moving along a straight line is given by $v=2t^3+6t^2$ where v is in m/sec and t is in seconds. Determine its acceleration and displacement at $t=5$ sec after it starts from origin.

or

c) Blocks A and B of mass 600kg and 1200kg respectively are connected by a string passing over a smooth pulley. Neglect mass of pulley. If coefficient of kinetic friction between the block A and the inclined surface is 0.2, determine the acceleration of block A and block B. Refer Fig. [6]

